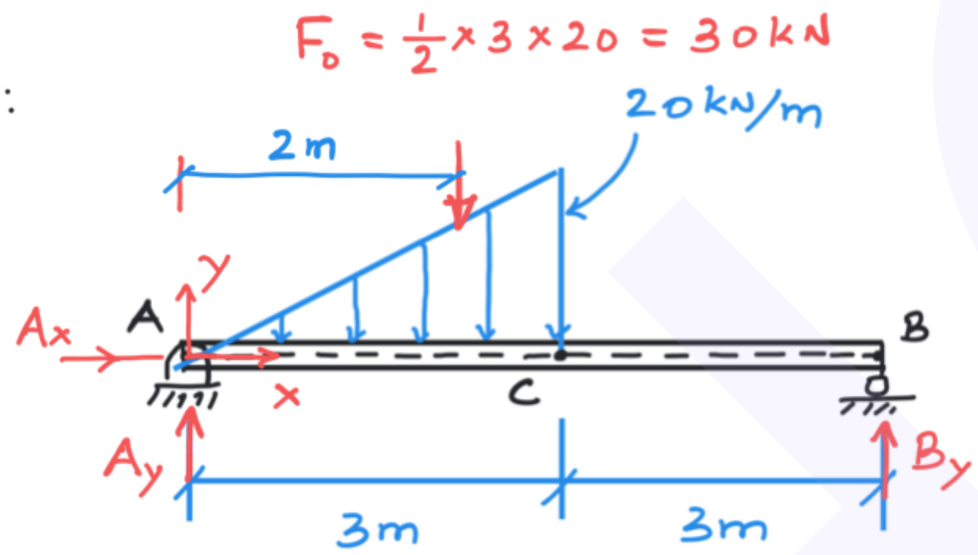
Question: Could you do an example on developing V & M diagrams for a beam by the mathematical function method? (Example 2: Distributed load)

Ans :

Problem :



Question: Draw the V & M diagrams

Solution :

Step 1 : Determine the support reactions.

$$\rightarrow \sum F_x = A_x = 0$$

$$\curvearrowleft \sum M_A = B_y \times 6 - 30 \times 2 = 0 \Rightarrow B_y = 10\,kN$$

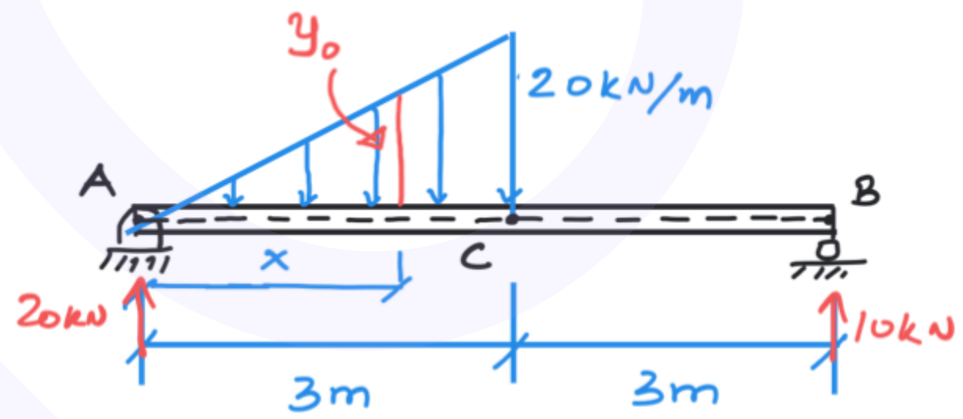
$$\uparrow \sum F_y = A_y + B_y - 30 = 0 \Rightarrow A_y = 30 - B_y = 30 - 10 = 20\,kN$$

Step 2: Identify points of discontinuity and segments to consider, and develop math functions for V & M in each segment.

* Points of discontinuity: A, B and C.

* Segments to consider: AC and CB

* Section in AC & consider the left side:



By similar triangles

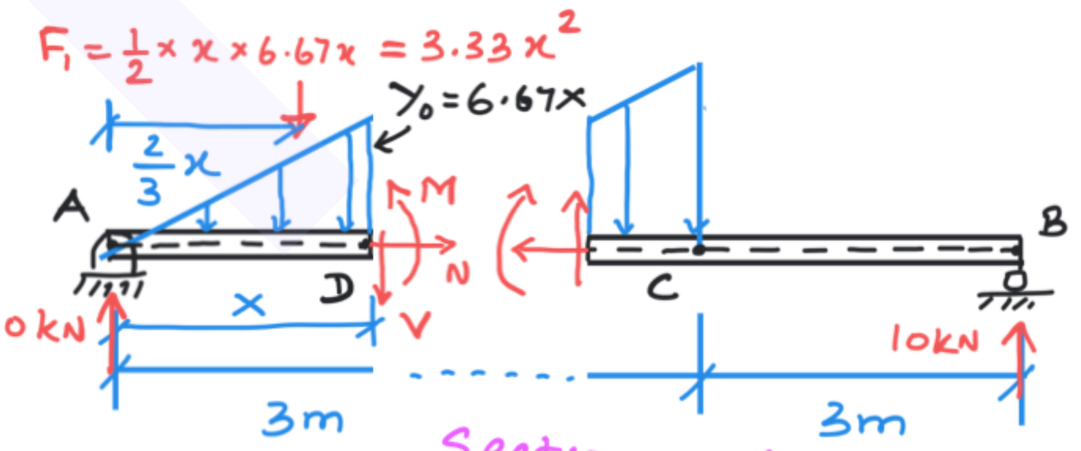
$$\frac{y_o}{x} = \frac{20}{3} \Rightarrow y_o = 6.667x$$

$$\uparrow \sum F_y = -V + 20 - 3.33x^2 = 0$$

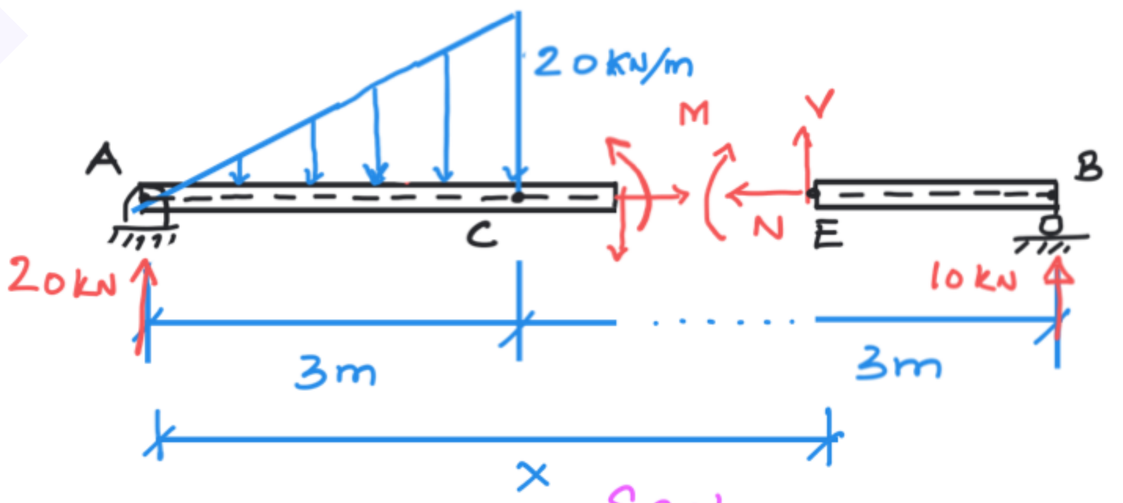
$$V = 20 - 3.33x^2 \quad \text{——— (1)}$$

$$\circlearrowleft \sum M_D = M + (3.33x^2)\left(\frac{x}{3}\right) - 20x = 0$$

$$M = 20x - 1.111x^3 \quad \text{——— (2)}$$



Section in AC



Section in CB

Step 2: Identify points of discontinuity and segments to consider, and develop math functions for V & M in each segment.

* Points of discontinuity: A, B and C.

* Segments to consider: AC and CB

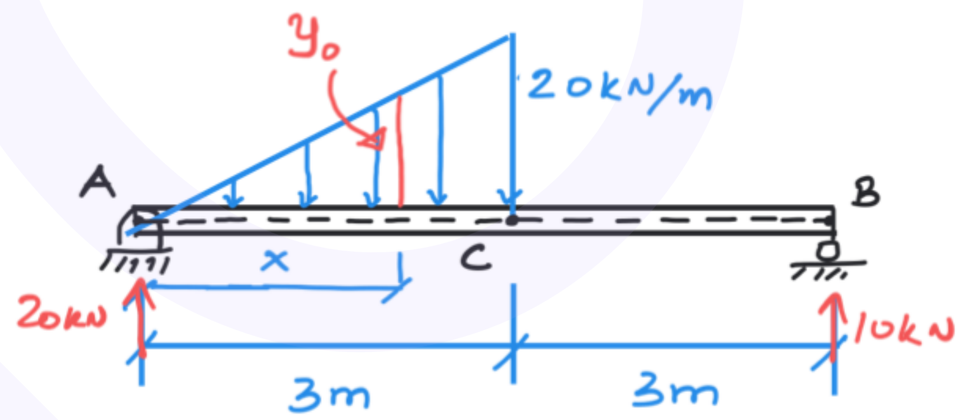
* Section in CB and consider the right side:

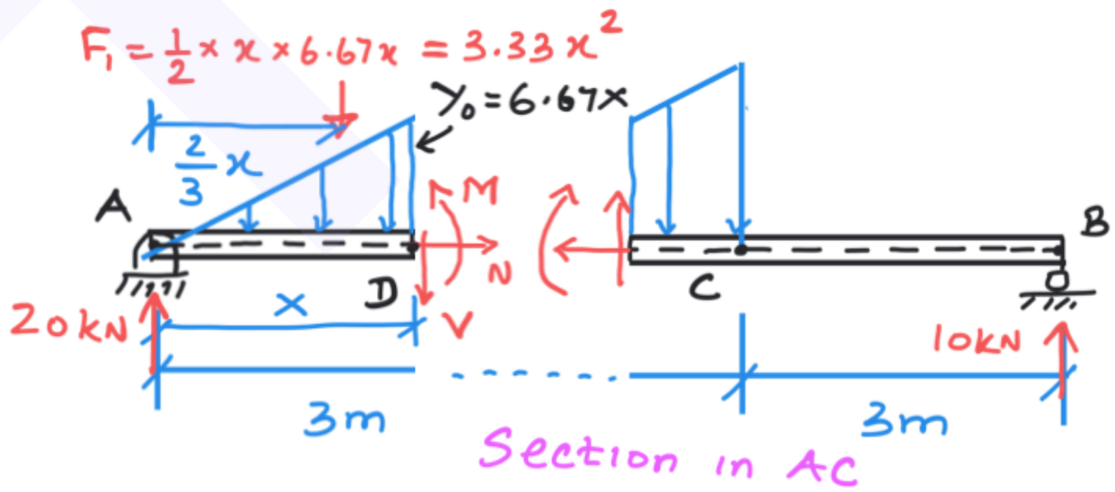
$$\uparrow \Sigma F_y = V + 10 = 0$$

$$V = -10 \text{ kN} \quad \text{——— (3)}$$

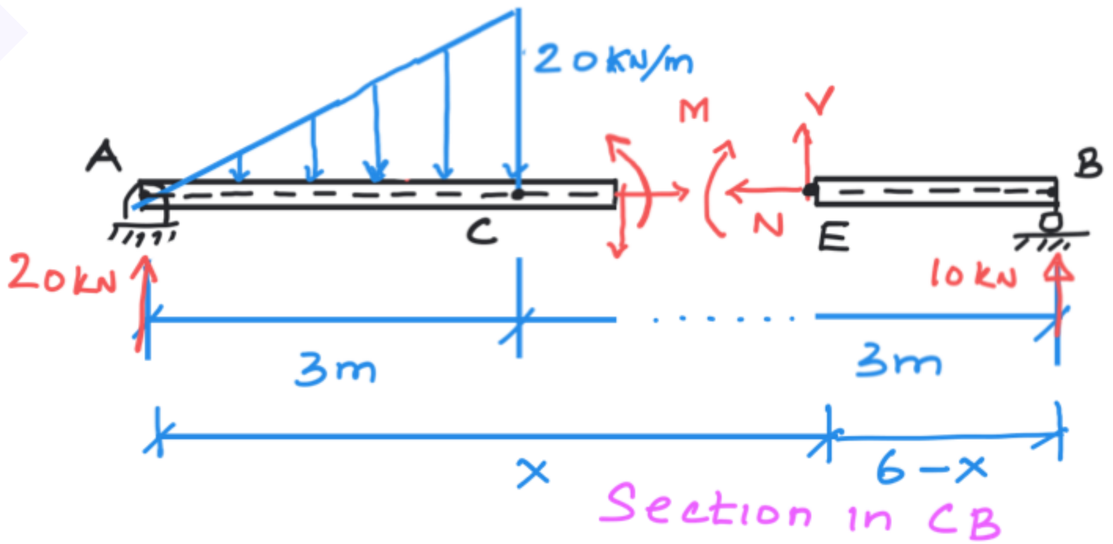
$$\curvearrowleft \Sigma M_E = -M + 10(6-x) = 0$$

$$M = -10x + 60 \quad \text{——— (4)}$$





Section in AC



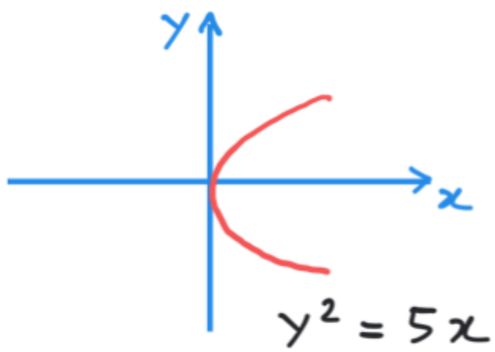
Section in CB

Step 3: Draw the V and M diagrams

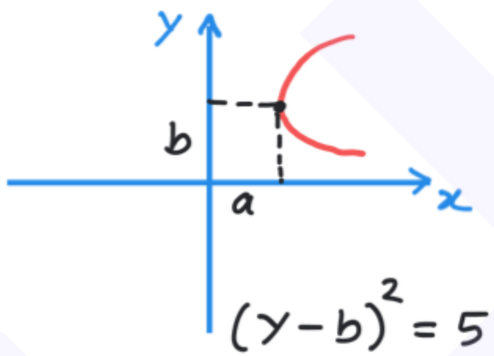
(a) Shear force

Segment AC: $V = 20 - 3.33x^2$ —— (1)

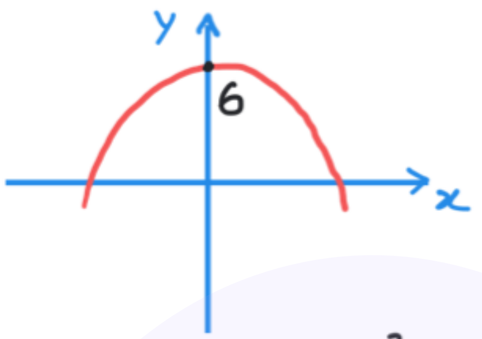
Recall: Parabolas



$y^2 = 5x$

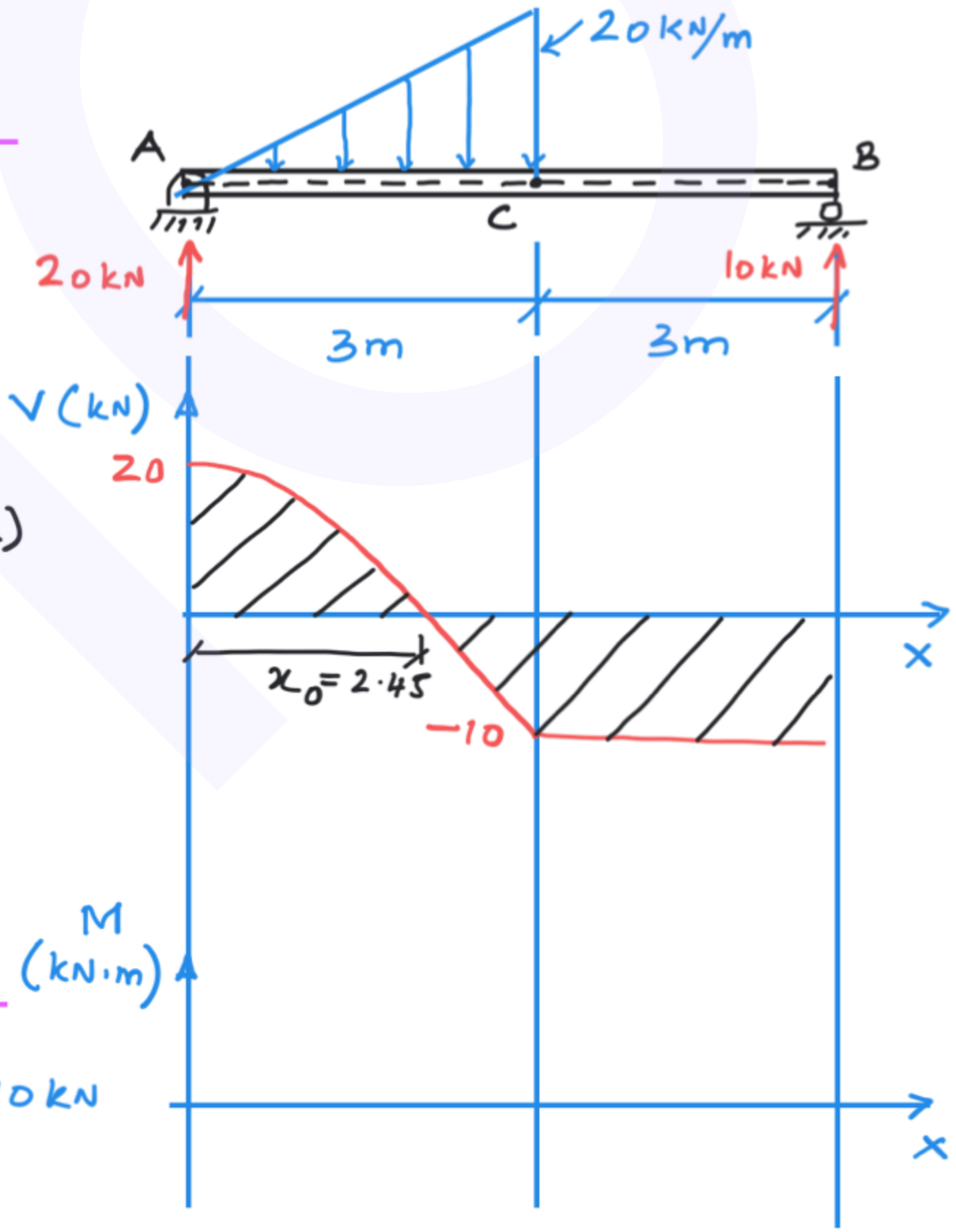


$(y-b)^2 = 5(x-a)$



$y - 6 = -5x^2$

$y = -5x^2$



From Eq (1): $x = 3m$, $V = 20 - 3.33 \times 3^2 = -10\,kN$

$V = 0 \Rightarrow x_0 = \sqrt{\frac{20}{3.33}} = 2.45m$

Segment CB: $V = -10\,kN$ —— (3)

Step 3: Draw the V and M diagrams

(b) Bending moment

Segment AC:

$$M = 20x - 1.111x^3 \quad \text{——— (2)}$$

$$\frac{dM}{dx} = 20 - 3.333x^2 \quad \text{——— (5)}$$

$x = 0$: $M = 0$

$$\frac{dM}{dx} = 20$$

$x = 3m$: $M = 20 \times 3 - 1.111 \times 3^3 = 30\ kN \cdot m$

$$\frac{dM}{dx} = 20 - 3.333 \times 3^2 = -10$$

Also note: $\frac{dM}{dx} = V$

When $V = 0$, $\frac{dM}{dx} = 0 \Rightarrow$ M is either maximum or minimum.

At $x = 2.45m$, $M = M_{max} = 20 \times 2.45 - 1.111 \times 2.45^3$

$= 32.7\ kN \cdot m$

Segment CB: $M = -10x + 60$ ——— (4)

$x = 3m$, $M = -10 \times 3 + 60 = 30\ kN \cdot m$

$x = 6m$, $M = -10 \times 6 + 60 = 0$